

Matrice

$$A = [a_{ij}]_{m \times n} \quad \begin{matrix} i = 1, 2, \dots, m \\ j = 1, 2, \dots, n \end{matrix}$$

Zadaci:

1. Zadane su matrice $A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ i $B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$. Nađi:

- a) $A + B$, b) $2A - 3B$, c) AB ,
d) BA , e) $AB - BA$,

2. Izračunati:

a) $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$, b) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, c) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$,

d) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, e) $\begin{bmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & 3 \\ 6 & 9 & 5 \end{bmatrix}$,

3. Izračunati:

a) $\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}^2$, b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^4$, c) $\begin{bmatrix} 4 & 3 \\ 7 & 5 \end{bmatrix} \cdot \begin{bmatrix} -28 & 93 \\ 38 & -126 \end{bmatrix} \cdot \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$.

4. Odrediti sve matrice koje komutiraju s danom matricom:

a) $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, b) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$.

Rezultate provjeriti!

5. Ako se zna da je $A \cdot A^{-1} = E$ nađi inverznu matricu sljedećih matrica:

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, b) $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, c) $A = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{bmatrix}$.

6. Izračunati $f(A)$ ako je $f(x) = 3x^2 - 2x + 5$ i $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{bmatrix}$.

7. Dokazati matematičkom indukcijom:

a) $\begin{bmatrix} 7 & 4 \\ -9 & -5 \end{bmatrix}^n = \begin{bmatrix} 6n+1 & 4n \\ -9n & 1-6n \end{bmatrix} \quad \forall n \in \mathbb{N}$

b) $\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}^n = \begin{bmatrix} 2n+1 & 2n \\ -2n & 1-2n \end{bmatrix} \quad \forall n \in \mathbb{N}$

8. Riješiti matricne jednačbe:

a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \cdot X = \begin{bmatrix} 4 & -6 \\ 2 & 1 \end{bmatrix},$

b) $X \cdot \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$

c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix},$

d) $\begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} \cdot X \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 14 & 16 \\ 9 & 10 \end{bmatrix}.$

9. Riješiti matricne jednačbe:

a) $(A - 2E) \cdot X = A + E,$

b) $X \cdot (A - 2E) = A + E,$

ako je $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}.$

10. Nađi sve matrice koje komutiraju s matricom:

a) $A = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix},$

b) $X = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}.$

11. Iskoristiti a) Cramerovo pravilo,
b) matricni postupak,
c) Gaussov postupak

u rješavanju sustava linearnih jednačbi:

$$2x + 3y + 5z = 10$$

a) $x + 2y + 2z = 3$

$$3x + 7y + 4z = 3$$



$$\begin{array}{rcl} 2x_1 & + & 2x_2 - x_3 - x_4 = 4 \\ 4x_1 & + & 3x_2 - x_3 + 2x_4 = 6 \\ \text{b)} & & 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12 \\ & & 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6 \end{array}$$

$$\begin{array}{rcl} & x_1 & - x_3 + x_4 = 3 \\ & 2x_1 + 3x_2 - x_3 - x_4 = 2 \\ \text{c)} & 5x_1 & - 3x_4 = 2 \\ & x_1 + x_2 + x_3 + x_4 = 2 \end{array}$$

$$\begin{array}{rcl} x & + & y - 2z = 1 \\ \text{d)} & -2x & + y + 4z = -2 \\ & -x & + 2y + kz = -1 \end{array} \quad \begin{array}{rcl} x & - & y - az = 1 \\ \text{e)} & x & + ay - z = 1 \\ & ax & + y - z = 1 \end{array}$$

[u primjerima d) i e) diskutiraj rješenja]

12. Riješiti homogene sustave:

$$\begin{array}{rcl} 2x & + & 3y - z = 0 \\ \text{a)} & x & + 5y + 2z = 0 \\ & 3x & + 8y + z = 0 \end{array} \quad \begin{array}{rcl} ax & - & y + 2z = 0 \\ \text{b)} & 3x & + (2a+3)y - z = 0 \\ & 4x & + 4y + z = 0 \end{array}$$

13. Naći zajedničko rješenje sustava:

$$\begin{array}{rcl} 3x & + & 2y = 5 \\ x & + & y = 2 \\ \underline{2x - y = 1} \end{array}$$

14. Odrediti parametar a tako da sustav ima rješenje i zatim naći to rješenje.

$$\begin{array}{rcl} 3x & + & ay = 5 \\ x & + & y = 2 \\ \underline{ax + 2y = 4} \end{array}$$

15. Za koje a sustav ima netrivialna rješenja. Za dobivene vrijednosti od a riješiti sustav:

$$\begin{array}{rcl} 2x & + & 6y + (a+6)z = 0 \\ -x & + & 7y + 5z = 0 \\ \underline{ax + 5y + 13z = 0} \end{array}$$